

Assignment 5

Logic

1. Is the following sentence valid, satisfiable or unsatisfiable?

$$C \vee (\neg A \wedge \neg B) \Leftrightarrow (A \Rightarrow C) \wedge (B \Rightarrow C)$$

2. Which of the following are entailed by the sentence $(P \vee Q) \wedge (\neg R \vee \neg S \vee \neg T)$?

- $(P \vee Q)$
- $(P \vee Q \vee R) \wedge (Q \wedge R \wedge S \Rightarrow T)$
- $(P \vee Q) \wedge (\neg S \vee T)$

3. Which of the following are semantically and syntactically correct translations of “Everyone’s ID number at NMSU has the same prefix”?

$$\forall x, d, ia (Dept(d) \wedge MemberOf(x, d) \wedge ID(x) = ia) \Rightarrow$$

- a. $(\forall y, ib MemberOf(y, d) \wedge ID(y) = ib \Rightarrow Prefix(ia) = Prefix(ib))$

$$\forall x, d (Dept(d) \wedge MemberOf(x, d) \wedge \exists ia ID(x) = ia) \Rightarrow$$

- b. $(\forall y, ib MemberOf(y, d) \wedge ID(y) = ib \wedge Prefix(ia) = Prefix(ib))$

c. $\forall x, y, d Dept(d) \wedge MemberOf(x, d) \wedge MemberOf(y, d) \Rightarrow Prefix(ID(x)) = Prefix(ID(y))$

d. $\forall x, y, d Dept(d) \wedge MemberOf(x, d) \wedge MemberOf(y, d) \Rightarrow Prefix(ID(x)) = Prefix(ID(y))$

4. Translate into FOL the sentence “Everyone’s eye color is unique and is derived from their parent’s eye color.” You must specify the exact meaning of your vocabulary terms. HINT: Do not use a predicate $Unique(x)$ since uniqueness is not a property of an object in itself.

5. Suppose a FOL knowledge base contains just the following Horn clauses:

$$Ancestor(Father(x), x)$$

$$Ancestor(x, y) \wedge Ancestor(y, z) \Rightarrow Ancestor(x, z)$$

Consider a forward chaining algorithm that, on the j th iteration, terminates if the KB contains a sentence that unifies with the query, else adds to the KB every atomic sentence that can be inferred from the sentences already in the KB after iteration $j - 1$.

- a. For each of the following queries, say whether the algorithm will (1) give an answer (if so, write that answer) or (2) terminate with no answer, or (3) never terminate.

i. $Ancestor(Father(y), John)$

ii. $Ancestor(Father(Father(y)), John)$

iii. $Ancestor(Father(Father(Father(y))), Father(y))$

iv. $Ancestor(Father(Father(John)), Father(John))$

b. Can a resolution algorithm prove from the original KB that $\neg Ancestor(John, John)$?

Explain your answer.

6. Show that the sentences:

a. $\forall x (\forall y P(x, y)) \Rightarrow Q(x)$

b. $\forall x \exists y (P(x, y) \Rightarrow Q(x))$

are logically equivalent by converting them to CNF. Give English sentences that interpret P and Q to make the sentences true in the real world.

Due date

March 31st before midnight. Please submit your answers through the file submitter in Word format (doc, docx), Open Office or PDF.